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(Affiliated to CBSE up to +2 Level)

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Real Numbers Class 10 Notes: Chapter 1

Question 1.Use Euclid's Division Algorithm to find the HCF of:

(i) 135 and 225 (ii) 196 and 38220 (iii) 867 and 255

Solution:

(ii) By Euclid's Division Algorithm, we have

38220 = 196 x 195 + 0

196 = 196 x 1 + 0

∴ HCF (38220, 196) = 196.

(iii) By Euclid's Division Algorithm, we have

867 = 255 x 3 + 102

255 = 102 x 2 + 51

102 = 51 x 2 + 0

∴ HCF (867, 255) = 51.

Question 2.Show that any positive odd integer is of the form 6q + 1, or 6q + 3, or 6q + 5, where q is some integer.

Solution

Let a be a positive odd integer. Also, let q be the quotient and r the remainder after dividing a by 6.

Then, a = 6q + r, where $0 \leq r < 6$.

Putting r = 0, 1, 2, 3, 4, and 5, we get:

a = 6q, a = 6q + 1, a = 6q + 2, a = 6q + 3, a = 6q + 4, a = 6q + 5

But a = 6q, a = 6q + 2 and a = 6q + 4 are even.

Hence, when a is odd, it is of the form 6q + 1, 6q + 3, and 6q + 5 for some integer q.

Hence proved.

Question 3.An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Solution:

Maximum number of columns = HCF of (616, 32)

For finding the HCF we should apply Euclid's division algorithm

Given numbers are 616 and 32

On applying Euclid's division algorithm, we have

616 = 32 x 19 + 8

Since the remainder 8 ≠ 0, so again we apply Euclid's division algorithm to 32 and 8, to get

32 = 8 x 4 + 0

32) 616 (19
 −32
 296
 −288
 8

8) 32 (4
 −32
 0

The remainder has now become zero, so we stop,

∵ At the last stage, the divisor is 8

∴ The HCF of 616 and 32 is 8.

Therefore, the maximum number of columns in which an army contingent of 616 members can march behind an army band of 32 members in a parade is 8.

Question 4.Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

Solution:

Let a be a positive integer, q be the quotient and r be the remainder.
Dividing a by 3 using the Euclid's Division Lemma,
we have:
$a = 3q + r$, where $0 \le r < 3$
Putting r = 0, 1 and 2, we get:
$a = 3q$
$\Rightarrow a^2 = 9q^2$
$= 3 \times 3q^2$
$= 3m$ (Assuming $m = q^2$)
Then, $a = 3q + 1$
$\Rightarrow a^2 = (3q + l)^2 = 9q^2 + 6q + 1$
$= 3(3q^2 + 2q) + 1$
$= 3m + 1$ (Assuming $m = 3q^2 + 2q$)
Next, $a = 3q + 2$
$\Rightarrow a^2 = (3q + 2)^2 = 9q^2 + 12q + 4$
$= 9q^2 + 12q + 3 + 1$
$= 3(3q^2 + 4q + 1) + 1$
$= 3m + 1$. (Assuming $m = 3q^2 + 4q+l$)
Therefore, the square of any positive integer (say, a^2) is always of the form 3m or 3m + 1.
Hence, proved.

Question 5.Use Euclid's Division Lemma to show that the cube of any positive integer is either of the form 9m, 9m + 1 or 9m + 8.

Solution:

Let 'a' be any positive integer and b = 3.
∴ By Euclid's division algorithm, we have $a = bq + r, 0 \le r \le b$
$a = 3q + r, 0 \le r < 3$ [∵ b = 3] where $q \ge 0$ and r = 0. 1, 2
∴ a = 3q or 3q + 1 or 3q + 2
Now

$$\begin{aligned} a^3 &= (3q)^3 \\ &= 27q^3 \\ &= 9(3q^3) \\ &= 9n, \text{ where } n = 3q^3 \end{aligned}$$

$$\begin{aligned} a^3 &= (3q+1)^3 \\ &= 27q^3 + 27q^2 + 9q + 1 \\ &= 9(3q^3 + 3q^2 + q) + 1 \\ &= 9t + 1, \\ &\text{where } t = 3q^3 + 3q^2 + q \end{aligned}$$

$$\begin{aligned} a^3 &= (3q+2)^3 \\ &= 27q^3 + 54q^2 + 36q + 8 \\ &= 9(3q^3 + 6q^2 + 4q) + 8 \\ &= 9k + 8, \\ &\text{where } k = 3q^3 + 6q^2 + 4q \end{aligned}$$

Thus, the cube of any positive integer is of the form 9m, 9m + 1 or 9m + 8.